MOAA 2025: Speed Round

October 11th, 2025

Rules

- You have 20 minutes to complete 10 problems. Each answer is a nonnegative integer no greater than 1,000,000.
- If m and n are relatively prime, then the greatest common divisor of m and n is 1.
- No mathematical texts, notes, or online resources of any kind are permitted. Rely on your brain!
- Compasses, protractors, rulers, straightedges, graph paper, blank scratch paper, and writing implements are generally permitted, so long as they are not designed to give an unfair advantage.
- No computational aids (including but not limited to calculators, phones, calculator watches, and computer programs) are permitted on any portion of the MOAA.
- No individual may receive help from any other person, including members of their team. Consulting any other individual is grounds for disqualification.

How to Compete

- **In Person:** After completing the test, write your answers down in the provided Speed Round answer sheet. The proctors will collect your answer sheets immediately after the test ends.
- **Online:** Log into the Classtime session to access the test. Input all answers directly into the provided form. Select for the test to be handed in once you are ready.

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Speed Round Problems

S1. **[2]** Compute $20 \times 25 + \sqrt{2025} + 2025$.

S2. **[2]** Let $ABCD$ be a square. Let M be the midpoint of CD, and N be the midpoint of AM. Given that the area of triangle BMN is 2025, find the length of AB.

S3. **[2]** Find the number of ordered pairs (m, n) such that $m + n = 2025$ and $\gcd(m, n) = 1$.

S4. **[3]** At Areteem, Eddie and Eugenia simultaneously begin reading the same book. Eddie reads 12 pages on day 1, and on each subsequent day he reads 3 more pages than the previous day. Eugenia reads 3 pages on day 1, and on each subsequent day she reads 5 more pages than the previous day. At the end of day M, they both have 1 page left to read. If the book contains N pages, find $N + M$.

S5. **[3]** Let S be the set of all positive integers n such that n and n^2 both end in the same three-digit number $\underline{a}\,\underline{b}\,\underline{c}$, with $a > 0$. Compute the fifth smallest number in S.

S6. **[4]** In regular heptagon $\mathcal{H}$, three pairs of vertices are randomly selected, and a line is drawn between every pair. The probability that the three lines drawn bound a triangle with a positive finite area can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m + n$.

Note: the same pair of vertices can be selected multiple times. For example, one could draw lines AB, AB, and AC.

S7. **[5]** Square $ABCD$ has side length 45. Let E be the midpoint of AB and let F be the midpoint of EB. Lines CE and CF intersect line BD at G and H, respectively. Let P be a point on line DA. Lines CE and CF intersect line BP at X and Y, respectively. Suppose $XY = 2YB$. Find the area of triangle PGH.

S8. **[6]** Find the number of ways to label the squares in a 6×6 grid satisfying the following conditions:

a) Each square contains exactly one number, which is in the set $\{1, 2, 3, 6\}$.

b) The numbers within each row and column multiply to 6.

c) Each row and column contains no more than two numbers greater than 1.

S9. **[6]** Find the number of nonnegative real numbers x satisfying the equation

$$\lfloor x \rfloor (x - \lfloor x \rfloor) = \frac{1}{2025} x^2.$$

Note: $\lfloor x \rfloor$ denotes the largest integer less than or equal to x.

S10. **[7]** Let ABC be an isosceles triangle with $AB = AC$, where the length of AB is an integer. Let point P lie on side AB and Q on side AC such that $AP = 17$, $AQ = 11$, and $\angle PBC = \angle PQC$. Given the length of CP is a positive integer, find the sum of all the possible lengths of CP.